STRESS AND STRAIN

Stress: force per unit area $\begin{cases} \text{tensile} & \sigma > 0 \\ \text{compressive} & \sigma < 0 \end{cases}$

$$\text{Engineering stress, } \sigma \equiv \frac{F}{A_o} \quad \begin{matrix} \leftarrow \text{load or force} \\ \leftarrow \text{orig. x-sec'nal area} \end{matrix}$$

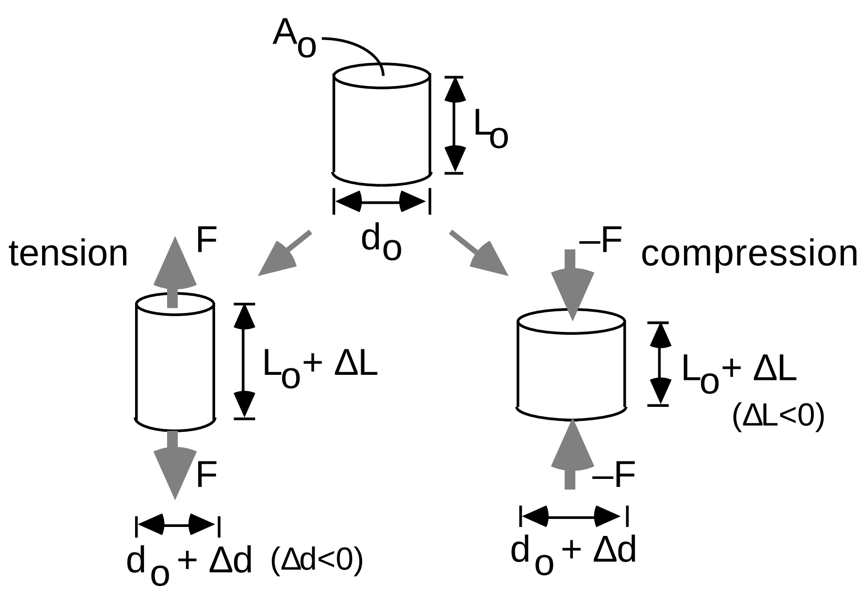
Units: Pascals $\left(\frac{N}{m^2}\right)$ or psi $\left(\frac{lbs}{in^2}\right)$

1 MPa = 10^6 Pa = 145 psi; 100 ksi (=100,000 psi) 700 MPa

Strain: change in length per unit length

$$\text{Engineering strain, } \varepsilon \equiv \frac{\Delta L}{L_o} \quad \begin{matrix} \leftarrow \text{change in length} \\ \leftarrow \text{original length} \end{matrix}$$

Units: *dimensionless*



Callister Figure 6.1

NOTE: strains parallel ($\varepsilon_{||}$) & *perpendicular* ($\varepsilon = \frac{\Delta d}{d_o}$) to load

Poisson's ratio

$$\nu \equiv \frac{-\varepsilon}{\varepsilon_{||}}$$

Usually, $0.16 \leq \nu \leq 0.36$

TENSILE TESTING MACHINE

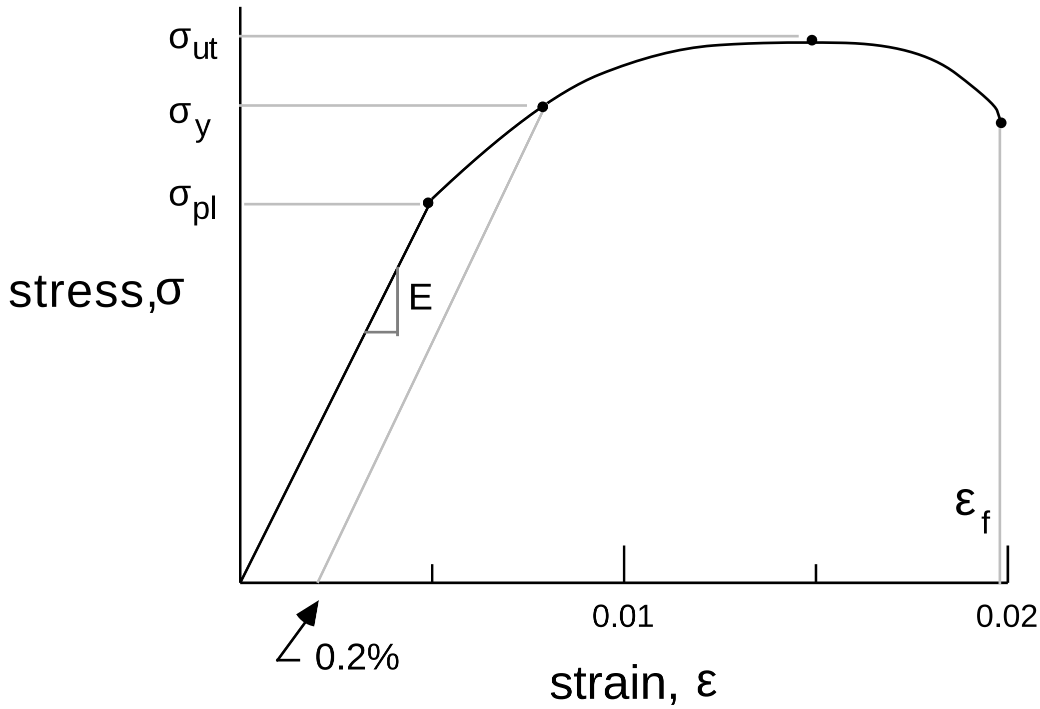
Callister, Fig. 6.3 — tensile testing machine

- "505 bar" — Nickname for the standard specimen most commonly used in tensile testing;
  a cylindrical specimen, 0.505" dia. along 2" gauge length (*i.e.,* the length of the straight section between threaded ends).
  This diameter gives a convenient cross-sectional area of 0.200 in^2.

- Measuring strain directly on the specimen with a strain gauge or extensometer, as opposed to using the nominal displacement of the crossheads, avoids the error in values of strain that would be associated with flexing of the support structure (crossheads, drive screws, *et c.*)

- Two categories of machines are available:
  - Screw-driven: allows selection and control of the *strain rate* ($d\varepsilon/dt$)
  - Hydraulically driven: allows selection and control of the *loading rate* ($d\sigma/dt$)

SCHEMATIC σ-ε CURVE for a DUCTILE METAL in TENSION



The following properties are defined with reference to a material's tensile stress-strain curve:

σ_{pl} — ***proportional limit*** — maximum stress in linear region

σ_y — ***yield strength*** or ***offset yield strength*** — stress that results in a specified amount of permanent strain (usu. 0.1% or 0.2%)

σ_{ut} — ***tensile strength*** or ***ultimate tensile strength*** — maximum (engineering) stress on curve

ε_f — ***elongation*** or ***strain to failure*** — total strain at break

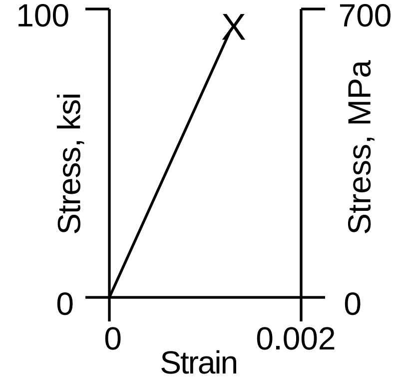
The tensile stress-strain curve illustrates both…

Elastic *behavior* — deformation is *reversible* (*i.e.*, $L \rightarrow L_o$ when $F \rightarrow 0$)

Plastic *behavior* — *permanent* strain occurs

ELASTIC BEHAVIOR ($\sigma < \sigma_{pl}$)

- *Reversible* or *recoverable* deformation ($\varepsilon \rightarrow 0$ when $\sigma \rightarrow 0$)
- Usually associated with *linear* region of σ-ε curve
- Hooke's law: $\sigma = E\varepsilon$, where the slope $\left(\frac{d\sigma}{d\varepsilon}\right) \equiv E$, the ***modulus of elasticity*** or ***Young's modulus***
    - In the linear region, $E \neq E(\sigma)$
    - E is a measure of…
        - Intrinsic stiffness
        - Bond strength, on the atomic level
    - For single-phase (or mostly single-phase) materials, E is relatively insensitive to…
        - Microstructure (*e.g.*, grain size, inclusions)
        - Degree of plastic deformation
    - E decreases with increasing T
- Hooke's law holds for *small* ε (typically <~0.1-0.2%)
- Ceramics, at room T, are Hookean up to fracture:



YOUNG'S MODULUS & THE INTERATOMIC POTENTIAL

Callister Fig. 6.6 — see also Callister Fig. 2.8

PROPERTIES DESCRIBING PLASTIC BEHAVIOR ($\sigma > \sigma_{pl}$)

- Proportional limit, σ_{pl} …
    - Demarcates transition from elastic to plastic behavior
    - Detecting a deviation from linearity depends on sensitivity of equipment and is somewhat subjective

        Less frequently reported
        Useful as a concept
- Offset yield strength, σ_y …
    - Always > σ_{pl}
    - More objective than σ_{pl}
    - *Primary design parameter denoting onset of plasticity*
    - To determine from σ-ε curve …
        - Draw line, || elastic region, through $\varepsilon = 0.2\%$
        - Find σ at which this line crosses σ-ε curve
- Tensile strength, σ_{ut} …
    - Fracture will result if σ_{ut} is applied and maintained
    - Easy to measure frequently reported
    - For design, indicates a stress at or beyond “failure”

Callister Fig. 6.10 — σ-ε curve, with insets of necking

- $\sigma < \sigma_{pl}$: deformation ($\varepsilon_{||}$ *and* ε) is …
    - … ***uniform*** along gauge length
    - … ***elastic***
- $\sigma_{pl} < \sigma < \sigma_{ut}$: deformation ($\varepsilon_{||}$ *and* ε) is …
    - … ***uniform*** along gauge length
    - … ***plastic***
- $\sigma_{ut} < \sigma$: deformation ($\varepsilon_{||}$ *and* ε) is …
    - … ***nonuniform*** along gauge length
    - … ***plastic***
    - Localized reduction in area: ***necking*** (increased ε)
    - Neck is also region of increased $\varepsilon_{||}$
    - True stress in neck > engineering stress

THE σ-ε CURVE BEYOND σ_{ut}

Q. If loading continues after σ_{ut} has been reached, how can elongation continue if σ is decreasing?

A. ***Necking*** is occurring (Figure 6.10).

- Definition: localized, permanent elongation and reduction in area
- Engineering stress, $\sigma \equiv \frac{\text{applied load}}{\textbf{\textit{original}}\ \text{cross-sectional area}}$, decreases;

  but stress *in the neck region*, $\frac{\text{applied load}}{\textbf{\textit{local}}\ \text{cross-sectional area}}$, continues to increase because the area of the neck is decreasing rapidly
- Note:
  - Necking begins at σ_{ut}
  - Total reduction in area includes (small) elastic and plastic components prior to reaching σ_{ut}, as well as reduction in area from necking

- Measures of ***ductility***, the ability to deform plastically
  - Reduction in area, $\%AR \equiv \frac{A_o - A_f}{A_o}$
  - % elongation, %EL (a.k.a. ε_f)

Q. What is happening on the crystal-structure level to bring about these macroscopic phenomena?

A. ***Slip*** — the generation and movement of dislocations

ENERGY(WORK) OF MECHANICAL DEFORMATION

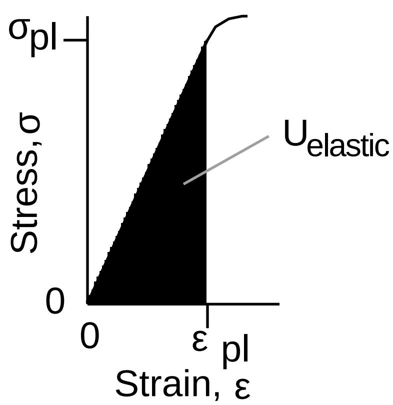
Area under the σ-ε curve gives $\frac{\text{energy}}{\text{volume}}$:

Work, $W \equiv \int_{L_o}^{L_i} F\, dL$ (where $L_i \equiv$ instantaneous length)

$$\frac{\text{energy}}{\text{volume}},\ U \equiv \frac{W}{V} = \frac{1}{AL}\int_{L_o}^{L_i} F\, dL = \int_0^{\varepsilon_i} \sigma\, d\varepsilon$$

- Total work of elastic deformation is a measure of ***resilience***

  Given: $\sigma = E\varepsilon$ $d\sigma = E d\varepsilon$ ($E \neq E(\sigma)$ in elastic region),



$$U_{elastic} = \int_0^{\varepsilon_{pl}} \sigma\, d\varepsilon = \int_0^{\sigma_{pl}} \frac{\sigma\, d\sigma}{E}$$

$$= \frac{\sigma_{pl}^2}{2E} \qquad \frac{\sigma_y^2}{2E}$$

modulus of resilience, $\boxed{U_r \equiv \frac{\sigma_y^2}{2E}}$

- Total work to fracture is a measure of ***toughness***
  - Given by total area under curve, U_{total}
  - Approximated by ***modulus of toughness,*** U_t:

  U_{total} $\boxed{\sigma_{ut}\varepsilon_f\ (\equiv U_t)}$ or U_{total} $\boxed{\frac{\sigma_y + \sigma_{ut}}{2} \times \varepsilon_f\ (\equiv U_t')}$

MECHANICAL PROPERTIES FOR SEVERAL MATERIALS

Material	E GPa (10^6 psi)	σ_y MPa (10^3 psi)	σ_{ts} MPa (10^3 psi)	% elong.
Metals				
Steel (1020, annealed)	207 (30.0)	295 (42.8)	395 (57.3)	36.5
Steel (4140, annealed)	207 (30.0)	417 (60.5)	655 (95)	25.7
Al alloy (2024, annealed)	72.4 (10.5)	75 (11)	185 (27)	20
Cast iron (gray G3000)	90-113 (13-16)		207 (30)	—
Ceramics				
Silicon nitride (sintered)	304 (44)	414-650 (60-94)		<0.2
Alumina (polycr., 99.9%)	380 (55)	282-551 (41-80)		<0.2
SiO_2 glass	73 (10.6)	104 (15)		<0.2
Polymers				
Linear addition thermoplastics[a]	0.04-0.60	9-73 (1.3-11)	8-72 (1.2-11)	2-1200
Linear condensation thermoplastics[b]	0.25-0.60	55-83(8.0-12)	63-94 (9.1-14)	15-150
Thermosetting polymers[c]	0.5-1.6	28-90 (4-13)		1.5-6
Elastomers[d]	$2\text{-}10\times10^{-3}$ ($3\text{-}15\times10^{-4}$)	6.9-24.1 (1-3.5)		350-2000

a) *E.g.*, polyethylene, polyvinyl chloride, polypropylene, polymethylmethacrylate, Teflon®

b) *E.g.,* nylon 6,6; polycarbonate

c) *E.g.*, phenolics, thermosetting polyesters, epoxies

d) *E.g.*, butadiene-acrylonitrile, styrene-butadiene, silicone

